Graph theory
Midterm exam
2013.10.21.

1. Define the following notions:
- a) trail   (12 points)
- b) cycle   (12 points)
- c) connected component   (12 points)
- d) crossing number   (12 points)
- e) chromatic number   (12 points)

2. Formulate the following theorems:
- a) Euler's formula   (20 points)
- b) Kuratowski's theorem   (20 points)

3. Prove Ore's theorem.   (40 points)

4. Find the chromatic index (edge-chromatic number) of the double 5-cycle (that is, the graph obtained by substituting every edge of the 5-cycle with two parallel edges).   (40 points)

5. Let the vertices of a graph G be the integers $1, 2, \ldots, 100$. The numbers $i \neq j$ are connected if they are *not* relatively prime numbers. Find the chromatic number of G.   (40 points)

6. There are seven knights on a chessboard, each of them can capture at least two of the others. Show that (at least) one of them can capture at least three of the others.   (40 points)

7. The degree sequence of a simple graph G is 10,7,7,7,7,7,7,7,7,5,5. Show that G is Hamiltonian.   (40 points)

8. Let G be a plane graph that has an Euler tour. Prove that its dual is bipartite.   (40 extra points)