Part I – 50 minutes

1. Build a $SHIQ$ TBox, representing the statements below. You can only use the concept names shown in grotesque font, and the role names $hasChild$ and $hasFriend$.

A Person is Lucky if she or he has a Happy grandparent and has at least two Clever children. We know that a Lucky Person either has no friends or all her/his parents are not Happy. We also know that no one can be both Clever and Lucky.

2. Consider the $SH$ TBox $T = \{O ⊑ ∃hC - O, ∃hF - O ⊑ B, hC - ∅ ⊑ hF\}$ and the reasoning task of deciding the satisfiability of concept $B$ wrt. $T$.

Write down the concept $C'$ and the TBox $T'$ obtained by the internalisation of this reasoning task, i.e. reduce this task to deciding the satisfiability of concept $C'$ wrt. TBox $T'$, where $T'$ contains role axioms only.

3. Transform the following concept $C_0$ into an equivalent concept $C_1$ in NNF:

$$C_0 = \neg((\geq 1 R) \land \forall R. (\neg B \sqcup \exists R. \neg B) \land \exists R.(\leq 5 R) \land (\geq 2 R.B)).$$

Part II – 30 minutes

5. Consider the following tableau state $T$, which was obtained in the process of deciding the satisfiability of the concept $C_0 = \exists hC.\exists hC.O \land \exists hC.\exists hC.B \land (\geq 2 hC) \land \forall hC.(\leq 1 hC) \land ((\exists hC.(\geq 2 hC)) \sqcup (\leq 1 hC))$:

$$b \cdot \{C_0, \exists hC.\exists hC.O, \exists hC.\exists hC.B, (\geq 2 hC), \forall hC.(\leq 1 hC), (\exists hC.(\geq 2 hC)) \sqcup (\leq 1 hC)\}$$

Which transformation rules of the $ALCN$ tableau algorithm for empty TBoxes are applicable in tableau $T$? For each applicable rule

• give the node(s) and the concept it applies to;
• construct its output, the set of tableau states $S_T$;
• check if any of the new states contains a clash.

(Note that you only have to deal with tableau states reachable from $T$ by a single rule application.)

When drawing tableau states, you don’t have to copy the unchanged node and edge labels. You can refer to a list of concepts in a node label of $T$ by . . . , i.e. when a rule extends a node label by a concept $D$ you can use the node label $\{\ldots, D\}$. 