Quantum Probability and Quantum Logic

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Instructor(s):

Mihály Weiner

Short Description of the Course:
The course is about the non-classical calculus of probability which is behind Quantum Physics. (Read this short summary written in a "Q&A" form about the essence of Quantum Physics.) The emphasis will be on the mathematical, information-theoretical and philosophycal aspects (but not directly on physics). In the first part of the course the necessary mathematicals tools are introduced, while in the second part, after describing a simple quantum physical system, we consider concepts like entanglement, we treat some "paradoxes" (such as the "EPR" paradox), overview some quantum information technological protocols (such as dense coding) and briefly discuss quantum computers.

Aim of the Course:
Information technological applications like quantum computers are the main motivations behind our study. However, the aim of the course is "merely" to give a sound foundation to the understanding of quantum phenomena; so the mentioned applications, though treated to some extent, will not form the core of the course.

Prerequisites:
The course requires linear algebra and basics of classical probability theory.

Detailed Program and Class Schedule:

1. The non-distributive structure of events in the quantum word. Elements of a non-classical probability theory I.
2. Elements of a non-classical probability theory II: the convex structure of probability functions, intrinsic uncertainties versus lack of knowledge.
3. Linear algebra brush up: complementary subspaces and projections, linear maps and their matrices in different bases, trace, determinant, eigenvalues and eigenspaces.
4. Advanced linear algebra: spectral decomposition and spectral calculus (only in finite dimensions), complex vector spaces with scalar product, operator norm, the adjoint of an operator.
5. The word of linear operators: normal, self-adjoint, unitary, and positive operators and their spectral characterization. Spectral calculus of normal operators.
6. The convex cone of positive operators. Operator inequalities and trace inequalities. The body of density operators.
7. Quantum probability theory: Hilbert lattices and Gleason's theorem (without proof).

--- take home midterm ----

8. The general framework of quantum physics. Physical quantities as self-adjoint operators on Hilbert spaces.
9. Study of an actual physical system (the spin system). The EPR paradox.
10. Symmetries of a quantum physical system. Wigner's theorem (without proof). Active and passive symmetry transformations.
11. Composite (bipartite) systems. The role of tensorial products. Entanglement.
12. Measurements, information gaining and information content. Classical bits versus qbits.
13. Physical operations as completely positive maps. The no-cloning theorem.

14. Dense-coding. Quantum gates and quantum computers. Example of an algorithm for a quantum computer (either Grover's search algorithm or Shor's algorithm for factorizing integers).

---- final ----

Method of Instruction:
Lectures, quizes and homeworks.

HOMEWORKS:
normally there is one exercise per week; students can usually choose their exercises from a selection.

Sample exercises from the first part (mathematical background) of the course:
http://www.renyi.hu/~mweiner/sample_exercises1.pdf

Sample exercises from the second part (quantum phenomena and their information technological applications) of the course:
http://www.renyi.hu/~mweiner/sample_exercises2.pdf

QUIZES:
there is a short (5 to 10 minutes) quiz every week on the material of the last week.

Sample quiz from the first part of the course:
http://www.renyi.hu/~mweiner/sample_quiz.pdf

Finally, here is an example for an independent study written up by a student of a previous semester on the informational capacities of a quantum bit:
http://www.renyi.hu/~mweiner/qbit.pdf

Textbooks:
Quantum Probabilities and Quantum Logic - handouts

Instructors' bio:

Mihály Weiner (born 1976) received his M.Sc. in physics (2001) at the Eötvös University but then went on to do a PhD in mathematics (obtained in 2005) at the University of Rome Tor Vergata. He had postdoctoral positions and research grants at several different places including the Institute for Theoretical Physics in Göttingen, the Erwin Schrödinger Institute for Mathematical Physics, the University of Rome Tor Vergata and the Rényi Institute of Mathematics of the Hungarian Academy of Sciences. Currently he is an assistant professor at the Department of Analysis, Budapest University of Technology and Economics (BME) but also teaches at the Budapest Semesters in Mathematics (BSM). His main research fields are operator algebras and matrix analyses with direct motivations coming from quantum physics. His Erd?s number is 2. Apart from English and Hungarian, he kept courses in Italian, too. He also plays the violin.